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DIFFERENTIAL FERTILITY AND ECONOMIC DEVELOPMENT

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This paper considers differential fertility and analyzes how the fertility of people caught in poverty disturbs their escape from poverty. For escape from poverty, it is necessary that the average human capital stock exceed certain thresholds before the ratio of the number of poor to rich people increases more rapidly than the human capital level of rich people. Thus, the escape depends on a race between the accumulation of human capital by the rich and the accumulation of children by the poor. A high initial ratio of the number of poor to rich people would imply persistent poverty.

Keywords: Differential Fertility, Child Labor, Ratio of the Number of Rich to Poor People, Development, Poverty

1. INTRODUCTION

This paper considers differential fertility and explores how poor people can escape poverty. Educational investment by rich people enhances the accumulation of the average human capital stock, which is crucial to development.[1] However, the ratio of the number of poor to rich people rises because of differential fertility. The rise in the ratio makes it more difficult for the average human capital stock to increase. Thus, the escape from poverty depends on a race between the accumulation of human capital by rich people and the accumulation of children by poor people.

The main motivation for studying differential fertility and development in less developed economies is based on empirical observations. Some less developed economies, such as sub-Saharan economies, have stagnated, and both fertility and income inequality have remained high in those economies. Table 1 reports the averages of fertility, GDP growth, poverty, and education in less developed sub-Saharan economies between 1960 and 2010. Fertility rates are still quite high, whereas the decline over time is small. The population growth rates changed little during this period. Compared with East Asian and Pacific developing economies, the growth rates of GDP per capita are quite small. The percentages of the population living on less than 1.25 and 2 dollars a day are approximately 50% and 75%, respectively. This implies that many people have been caught in poverty. The

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TABLE 1. Fertility, GDP growth, poverty, and education in sub-Saharan economies

	1960s	1970s	1980s	1990s	2000s
Fertility rate	6.68	6.70	6.48	5.88	5.27
Population growth	2.50	2.77	2.84	2.66	2.51
Per capita GDP growth	2.45	0.93	−0.98	−0.37	2.21
Poverty headcount ratio ($1.25)			55.30	58.00	52.93
Poverty headcount ratio ($2)			74.85	77.13	74.22
Primary completion rate				51.08	58.73
Secondary school enrollment				19.08	23.51

Note: The years, represented as x, include 10 years from $x + 1$ to $x + 10$. The World Bank presents the cross-country average data of developing sub-Saharan economies. We calculate the averages over 10 years where possible. Population growth and per capita GDP growth represent the rates of growth. Poverty headcount ratios ($1.25) and ($2) represent the poverty head count ratios at $1.25 and $2 a day (% of population), respectively. Primary completion rate represents the primary completion rate (% of relevant age group). Secondary school enrollment represents the net secondary school enrollment rate.

primary school completion rates and secondary school enrollment rates remain low. Furthermore, high fertility is related closely to the existence of child labor. According to the International Labour Office (2010), 215 million child laborers are working worldwide. The sub-Saharan region accounts for 30% of child laborers in the world; 25% of all children in the region are laborers.

This paper investigates why it is difficult for less developed economies with high income inequality, such as sub-Saharan economies, to undergo a demographic transition that leads to further development. Rich and poor people exist in our model. They are homogenous, except for their initial human capital levels. Rich people, but not poor people are initially educated. Loans are assumed to be unavailable.[2] Educated and uneducated labor are, respectively, defined as skilled and unskilled labor. Because we assume that skilled and unskilled labor are not perfectly substitutable, their marginal products depend on their relative quantities. Although a rise in the amount of unskilled labor decreases its marginal productivity, a rise in the amount of skilled labor increases the marginal productivity of unskilled labor. If the marginal product of unskilled labor increases sufficiently, poor people can escape poverty by starting educational investment. The speeds of increases in the amounts of skilled and unskilled labor play a crucial role in the accumulation of the average human capital stock represented as the ratio of skilled labor to unskilled labor.

The average human capital stock depends on the human capital stock of rich people and the ratio of the number of rich to poor people. The ratio of the number of rich to poor people, which is dependent on the ratio in the previous period, declines because the fertility rate of the poor is always higher than that of the rich. Furthermore, this ratio asymptotically approaches zero unless poor people start educational investment. It is necessary that the average human capital stock exceed certain thresholds for an escape from poverty before an increase in the ratio of the number of poor to rich people (that is, the inverse of the ratio of the number of rich to poor people) outweighs the accumulation of human capital of the rich.

Two features in our model characterize less developed economies. The first feature is not only modern technology that requires both skilled and unskilled labor, but also traditional technology that requires only unskilled labor. When traditional technology remains in use, the unskilled labor wage does not increase, even with the accumulation of the average human capital stock. As a result, poor people can be caught in poverty. The second feature is child labor. Because child labor increases household income, the children of poor people are compelled to work. Thus, child labor increases the fertility rate of the poor, making it more difficult for the average human capital stock to increase.

The remainder of this paper is organized as follows. The next section places this paper in the context of the existing literature. Section 3 explains our model, and Section 4 presents a description of the development of an economy. We first examine the ratio of skilled labor to unskilled labor. We then consider two stages of development: the start of a rise in the poor's income level and the start of their educational investment. We conclude in Section 5 with a brief summary.

2. RELATED LITERATURE

Many studies have investigated the evolution of fertility, human capital, and income in the process of development and indicated the importance of the demographic transition to sustained economic growth. Galor and Weil (2000) and Galor and Moav (2002) examined the evolution of population, technology, and output and presented unified growth models that offer explanations for the takeoff from poverty to growth.[3] Considering the fertility differential between rich people and poor people, de la Croix and Doepke (2003) examined the relationship between inequality and growth and reported that different fertility behaviors of the rich and the poor account for most of the empirical relationship between inequality and growth.

Some studies have considered factors that disturb the demographic transition and development. Hazan and Berdugo (2002) investigated the effects of child labor on human capital accumulation and development, showing that, in the early stages of development, an economy may be in a poverty trap where child labor is abundant. Moav (2005) showed that if educated individuals have a comparative advantage in rearing educated children, a poverty trap could appear in which poor people choose a high fertility rate with a low level of investment in child quality.[4] This paper shows that a race between the accumulation of human capital by rich people and the accumulation of children by poor people plays a crucial role in the escape from poverty by the poor. If it is impossible for the average human capital stock to exceed certain thresholds before an increase in the ratio of the number of poor to rich people outweighs the accumulation of human capital by the rich, poverty will be persistent. Furthermore, although the existence of child labor disturbs the start of a rise in the poor's income level, its prior existence also has an adverse effect on the start of education investment by the poor.

This paper is also related to Maoz and Moav (1999). Assuming a complementary relationship between skilled and unskilled labor, they showed that intergenerational mobility is correlated positively with wage equality: mobility increases as an economy develops. This paper shows that, because the ratio of the number of poor to rich people increases as a result of differential fertility, a trickle-down effect might not occur even with the accumulation of human capital by the rich. As explained by Weil (2005), growth would be good for poor people. However, the fertility of the poor might disturb their escape from poverty.

3. MODEL

We consider a closed overlapping-generations economy. Individuals live for two periods. Parents choose their fertility rates. Additionally, parents decide whether their children work as laborers or receive education. Parents obtain income from child labor if children work in the first period. If parents decide to invest in education for their children, then the children can receive education in the first period. Children who do not receive education work as child labor in the first period, whereas they work as unskilled laborers in the second period. Children who receive education in the first period can work as skilled laborers in the second period. The initial education levels of rich people and poor people are denoted, respectively, by $e_{r,-1}$ and $e_{p,-1}$. We assume that $e_{r,-1} > 0$ and $e_{p,-1} = 0$. The population born in period t is L_t. We designate the ratio of rich people to the total population as λ_t.

3.1. Individuals

We first describe the relationship between educational investment and human capital formation. Although educational investment can raise the human capital levels of the children, for simplicity, we assume the following linear relationship:

$$h_{it} = 1 + q_{it} \gamma e_{it}, \tag{1}$$

where $i = r, p$. We assume that $0 < \gamma$. Here, h_{rt} and h_{pt} denote, respectively, the human capital stock levels of rich people and poor people formed in period t. q_{it} represents the decision with respect to work or education of their children, and takes a value of zero or unity. A value of zero implies that parents force their children to work, whereas a value of unity implies that parents compel their children to receive education. e_{rt} and e_{pt} denote, respectively, the educational levels of the children of rich people and poor people received in period t.

A decision represented as $q_{it} = 0$ implies that $e_{it} = 0$. Parents choose the educational level with a decision of $q_{it} = 1$. Equation (1) implies that the human capital level is still positive, even with no educational investment. Consequently, unskilled laborers can obtain enough income to live.

We define the income level for which individuals can work with no child rearing as the potential income level. The potential income level of an individual born in period $t-1$ is represented as

$$I_{it} = w_{jt} h_{it-1}, \tag{2}$$

where $i = r, p$ and $j = s, u$. I_{rt} and I_{pt} denote, respectively, the potential income levels of rich people and poor people in period t. w_{ut} and w_{st} denote the wage rates of unskilled and skilled labor, respectively. We assume that the wage rate of skilled labor exceeds that of unskilled labor; that is, $w_{st} > w_{ut}$.

Parents care about their consumption level, their fertility rate, and the education level of their children. They select the consumption level and the fertility rate. Moreover, they decide whether their children work as child laborers or receive education. They choose the educational level of their children, if they can afford education for their children. We consider the cost of child rearing an opportunity cost. The utility maximization problem of an individual born in period $t-1$ is written

$$\max_{q_{it}, c_{it}, n_{it}, e_{it}} \quad \beta \ln c_{it} + (1-\beta) \ln n_{it} (o + e_{it})^{\delta}, \tag{3}$$

$$\text{s.t.} \quad (1 - \eta n_{it}) I_{it} + (1 - q_{it}) n_{it} b w_{ut} = c_{it} + q_{it} n_{it} w_{st} e_{it}, \tag{4}$$

where $i = r, p$. We assume that $0 < o$, $0 < \eta < 1$, $0 < \delta < 1$, $0 < b < 1$, $0 < 1 - \eta n_{it} < 1$, and $0 < b < \eta$.[5] η represents the duration of child rearing per child. c_{rt} and c_{pt} denote the consumption levels of the rich and the poor, respectively. n_{rt} and n_{pt} denote the fertility rates of the rich and the poor, respectively.

Child labor increases the family income level. We assume that the income level obtained from child labor is less than the income level of unskilled labor; that is, $b < 1$. If parents force their children to receive education, they must pay education costs. However, their utility level increases with a rise in the education level. We allow a zero education expenditure by assuming o.[6] We assume that the cost of education is proportionate to the wage rate of skilled labor, because individuals who receive education can become skilled laborers.

The first-order conditions of the utility maximization problem imply that education investment is convex because of o. Parents cannot afford education for their children when the following condition holds:

$$\eta \delta I_{it} - o w_{st} \leq 0. \tag{5}$$

When the income level is low, parents force their children to work as child laborers; that is, they choose $q_{it} = 0$. We then have

$$e_{it} = 0, \tag{6}$$

$$n_{it} = \frac{1-\beta}{\eta - b w_{ut}/I_{it}}, \tag{7}$$

$$c_{it} = \beta I_{it}. \tag{8}$$

We compare the ratio of the marginal benefit from an additional child to the marginal cost with the ratio of the marginal benefit of additional education to the marginal cost at $e_{it} = 0$. The former is greater than the latter because of a positive human capital level under no education, child labor, and the low income level. Because parents can obtain income from child labor, the existence of child labor raises the fertility rate. Thus, the working hours, represented as $1 - \eta n_{it}$, decrease. The consumption level is proportionate to the potential income level, implying that the consumption level per working hour increases because of the income obtained from child labor.

Next, let us consider the case in which the potential income level of parents is high enough to satisfy the following expression:

$$\eta \delta I_{it} - o w_{st} > 0. \qquad \textbf{(9)}$$

Consequently, it is possible for parents to invest in education for their children.

Parents can attain the following utility level if they force their children to work:

$$U|_{q_{it}=0} \equiv \beta \ln I_{it} + (1-\beta) \ln \frac{1}{\eta - b w_{ut}/I_{it}} + B,$$

where

$$B \equiv \beta \ln \beta + (1-\beta) \ln (1-\beta) o^{\delta}.$$

If parents can afford education for their children, then they can attain the following utility level:

$$U|_{q_{it}=1} \equiv [\beta + (1-\beta)\delta] \ln I_{it}$$

$$+(1-\beta)(1-\delta) \ln \frac{1}{\eta - o w_{st}/I_{it}} - (1-\beta)\delta \ln w_{st} + C,$$

where

$$C \equiv \beta \ln \beta + (1-\beta) \ln (1-\beta)(1-\delta) + (1-\beta)\delta \ln \frac{\delta}{1-\delta}.$$

Parents have an incentive to invest in education for their children if the utility level represented by $U|_{q_{it}=1}$ is higher than the level represented by $U|_{q_{it}=0} = 0$. We assume the incentive-compatible condition, represented as[7]

Assumption 1

$$U|_{q_{it}=1} - U|_{q_{it}=0} > 0. \qquad \textbf{(10)}$$

When the potential income level is high enough to satisfy (9) and the incentive-compatible condition shown in (10) holds, parents will afford education for their children: they will choose $q_{it} = 1$. The first-order conditions are as follows:

$$e_{it} = \frac{\delta \eta I_{it}}{(1-\delta) w_{st}} - \frac{o}{1-\delta}, \qquad \textbf{(11)}$$

$$n_{it} = \frac{(1-\beta)(1-\delta)}{\eta - ow_{st}/I_{it}}, \tag{12}$$

$$c_{it} = \beta I_{it}. \tag{13}$$

Given the wage rate of skilled labor, the level of educational investment increases with the potential income level. Furthermore, the income elasticity for education investment is greater than unity. The fertility rate decreases as the income level increases, but this decrease becomes smaller as income continues to increase.[8] An increase in income raises the opportunity cost of child rearing, inducing parents to spend more on educational investment and to have fewer children. The burden of educational investment on household budgets increases with a rise in the potential income level because of the more-than-offsetting increase in educational investment per child. The consumption level is proportionate to the potential income level.

3.2. Firms

Firms are perfectly competitive. Two types of technology can be used: traditional and modern. A firm determines which type of technology should be used to minimize costs. A linear production function in which only unskilled labor is used is assumed for traditional technology:

$$Y_t = A_T l_{Tut}, \tag{14}$$

where we assume that $0 < A_T$. Y_t is the output in period t, and l_{Tut} is the input of unskilled labor and child labor for the traditional technology in period t. We assume that only traditional technology uses child labor.

Additionally, we assume a CES production function for modern technology. The CES production function can be represented as

$$Y_t = A_M[d l_{Mst}{}^{-\rho} + (1-d) l_{Mut}{}^{-\rho}]^{-1/\rho}, \tag{15}$$

where we assume that $0 < A_M$, $0 < d < 1$, and $-1 < \rho < \infty$.[9] l_{Mst} and l_{Mut} denote, respectively, the inputs of skilled and unskilled labor for modern technology in period t. $\sigma \equiv 1/(1+\rho)$ represents the elasticity of substitution between skilled and unskilled labor.

When the traditional technology represented in (14) is used, the wage rate of unskilled labor equals the shift parameter A_T:

$$A_T = w_{ut} \equiv w_{u0}. \tag{16}$$

Modern technology implies the following first-order conditions:

$$A_M[d + (1-d)(l_{Mst}/l_{Mut})^{\rho}]^{-(1+\rho)/\rho} d = w_{st}, \tag{17}$$

$$A_M[d(l_{Mst}/l_{Mut})^{-\rho} + (1-d)]^{-(1+\rho)/\rho}(1-d) = w_{ut}. \tag{18}$$

Two cases are possible at equilibrium: one in which both types of technology are applied indifferently and another in which only modern technology is chosen.[10]

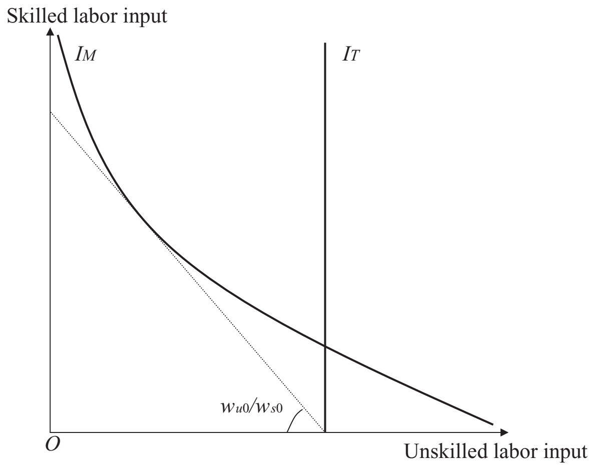

FIGURE 1. Isoquants of traditional technology and modern technology.

When both types of technology are used, as shown in (16), the wage rate of unskilled labor is constant. The wage rate of skilled labor is also constant, in which case the ratio of the skilled labor input to the unskilled labor input is given as

$$\frac{l_{Mst}}{l_{Mut}} = d^{1/\rho}\left[\left(\frac{1}{1-d}\frac{A_T}{A_M}\right)^{-\rho/(1+\rho)} - (1-d)\right]^{-1/\rho}. \qquad \textbf{(19)}$$

Figure 1 portrays the case in which both types of technology are used. For a given output level, I_T and I_M represent the isoquants of traditional technology and modern technology, respectively. If the ratio of the wage rates of unskilled labor to skilled labor is higher than w_{u0}/w_{s0}, then only modern technology is used.

4. HOW DOES DIFFERENTIAL FERTILITY AFFECT DEVELOPMENT?

4.1. The Ratio of Skilled Labor to Unskilled Labor

We first examine the ratio of skilled labor to unskilled labor. We define the ratio of skilled labor to unskilled labor in efficiency units in period t as H_t. We assume that this ratio is initially lower than the following threshold, represented as H_{I}:

$$H_0 < d^{1/\rho}\left[\left(\frac{1}{1-d}\frac{A_T}{A_M}\right)^{-\rho/(1+\rho)} - (1-d)\right]^{-1/\rho} \equiv H_{\text{I}}. \qquad \textbf{(20)}$$

As H_{I} is represented in (19), (20) implies that both types of technology are used.

Although poor people are initially unskilled laborers, rich people are skilled laborers. The inequality expressed in (5) is rewritten as

$$\delta\eta w_{u0} - o w_{s0} \leq 0. \qquad \textbf{(21)}$$

Equation (21) implies that, when the ratio of the wages of unskilled labor to the wages of skilled labor is low, poor people do not invest in education for their children. Poor parents force their children to work as child laborers: they choose $q_{pt} = 0$. The first-order conditions of their utility maximization imply that

$$e_{pt} = 0, \tag{22}$$

$$n_{pt} = \frac{1-\beta}{\eta - b} \equiv n_{pI}, \tag{23}$$

$$c_{pt} = \beta w_{u0}. \tag{24}$$

The human capital level of the poor remains constant in stage I; that is, $h_{pt} = 1$. Consequently, poor people continue to be unskilled laborers. The fertility rate takes a constant value because the cost of rearing children is proportionate to income.

Assumption 2

$$\delta \eta h_{r,-1} > o. \tag{25}$$

This assumption implies that (9) always holds when parents are skilled laborers. Consequently, the income level of the rich is high enough for them to afford education for their children.

The incentive-compatible condition in (10) can be rewritten as

$$(1-\beta)\ln\left(\eta h_{rt-1} - b\frac{w_{u0}}{w_{s0}}\right)\left(\frac{1}{\eta h_{rt-1} - o}\right)^{1-\delta} \delta^{\delta}(1-\delta)^{1-\delta} o^{-\delta} > 0. \tag{26}$$

The left-hand side increases with rises in the wage ratio of skilled labor to unskilled labor and the human capital level of rich people. That is, when the income gap between the rich and the poor is large, this incentive-compatible condition can hold.

When (25) and (26) hold, rich people who are skilled laborers always invest in education for their children; that is, they choose $q_{rt} = 1$. The first-order conditions of their utility maximization problem imply that

$$e_{rt} = \frac{\delta \eta h_{rt-1} - o}{1-\delta}, \tag{27}$$

$$n_{rt} = \frac{(1-\beta)(1-\delta)}{\eta - o/h_{rt-1}} \equiv n(h_{rt-1}), \tag{28}$$

$$c_{rt} = \beta h_{rt-1} w_{s0}. \tag{29}$$

As shown in Figure 2, when the human capital level increases, education investment increases and the fertility rate decreases.

Using (1) and (27), the human capital level of the rich can be expressed as follows:

$$h_{rt} = \frac{\gamma \delta \eta}{1-\delta} h_{rt-1} + \frac{1-\delta-\gamma o}{1-\delta} \equiv x(h_{rt-1}), \tag{30}$$

where we assume that $\gamma \delta \eta < 1 - \delta$ and $1 - \delta > \gamma o$.[11]

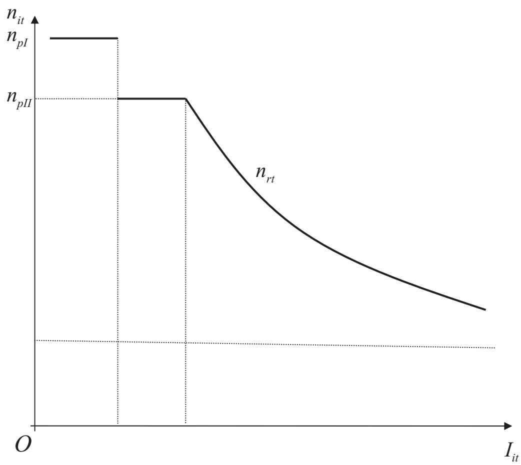

FIGURE 2. Fertility rate and income level.

Figure 3 shows the dynamics of the human capital level of the rich. There exists only a stable steady state. When individuals are initially skilled laborers, given the initial value satisfying (25), their human capital level necessarily converges to $h^* = (1 - \delta - \gamma o)/(1 - \delta - \gamma \delta \eta)$.

When traditional and modern technologies are viewed indifferently by firms, the factor market equilibrium conditions are written as

$$\mu_t l_{Mst} = (1 - \eta n(h_{rt-1}))h_{rt-1}\lambda_{t-1}L_{t-1}, \tag{31}$$

$$\mu_t l_{Mut} + (1 - \mu_t)l_{Tut} = (1 - \eta n_{pI} + bn_{pI})(1 - \lambda_{t-1})L_{t-1}, \tag{32}$$

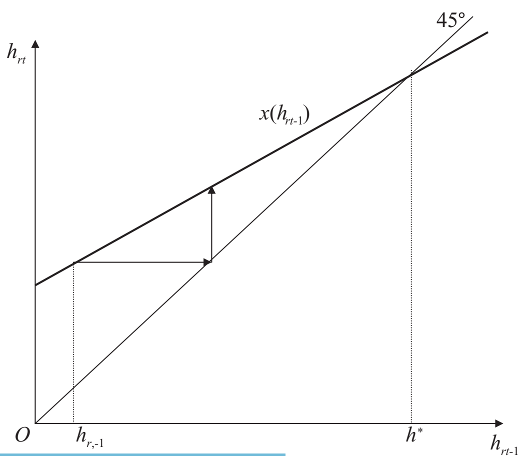

FIGURE 3. Dynamics of human capital level for rich people.

where, after the number of firms is normalized to unity, μ_t represents the number of firms using modern technology.

When the ratio of skilled labor to unskilled labor increases, the proportion of firms using modern technology increases. The ratio of skilled labor to unskilled labor measured in terms of efficiency units can be represented as

$$H_t = \frac{(1 - \eta n(h_{rt-1}))h_{rt-1}}{1 - \eta n_{pI} + bn_{pI}} \frac{\lambda_{t-1}}{1 - \lambda_{t-1}}. \tag{33}$$

The ratio of skilled labor to unskilled labor depends on the human capital level of rich people including their working hours and the ratio of the number of rich to poor people.

The ratio of rich people to the total population evolves as follows:[12]

$$\lambda_t = \frac{n_{rt} L_{rt-1}}{n_{rt} L_{rt-1} + n_{pt} L_{pt-1}} = \frac{n(h_{rt-1})\lambda_{t-1}}{n(h_{rt-1})\lambda_{t-1} + n_{pI}(1 - \lambda_{t-1})}. \tag{34}$$

As rich people accumulate human capital, the ratio of rich people to the total population decreases because $n_{pI} > n(h_{rt-1})$ holds. A high fertility rate for the poor decreases the ratio of rich people to the total population, and a decline in the fertility rate of the rich further decreases the ratio.

Using (34), the ratio of the number of poor to rich people can be written as

$$\frac{1 - \lambda_t}{\lambda_t} = \frac{n_{p\mathrm{I}}}{n(h_{rt-1})} \frac{1 - \lambda_{t-1}}{\lambda_{t-1}}. \tag{35}$$

The dynamics of the ratio of the number of poor to rich people depends on its value in the previous period and the coefficient represented by the ratio of fertility rates of the poor to the rich. Although this coefficient always exceeds unity, it increases with the accumulation of human capital by the rich. Consequently, the speed of rises in the ratio of the number of poor to rich people can increase, along with the human capital accumulation of the rich. This ratio asymptotically approaches infinity unless the poor start educational investment. Thus, (33) implies that an increase in the ratio of skilled labor to unskilled labor depends on the race between the accumulation of human capital by the rich and the accumulation of children by the poor.

4.2. The Start of a Rise in the Poor's Income Level

We now investigate whether the poor's income level can start to rise. The poor's income level starts to rise if the following condition holds:

$$H_{t+1} > H_{\mathrm{I}}. \tag{36}$$

We denote the stage before the poor's income level begins to rise as stage I.

We examine (36) in two ways. We first examine the change in the ratio of skilled labor to unskilled labor directly in order to present an intuitive explanation. Using

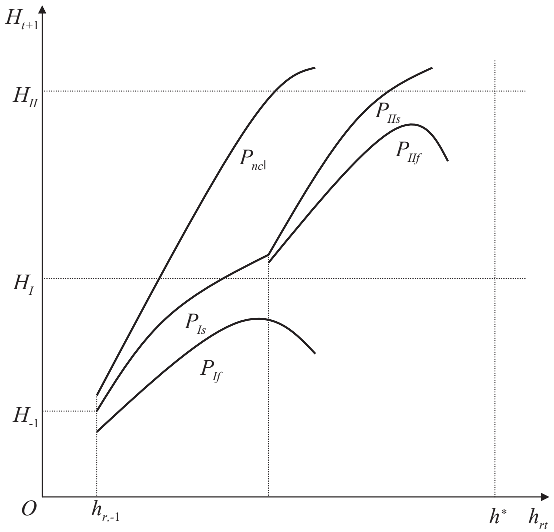

FIGURE 4. Conditions represented as (36) and (43).

(33) and (34), this growth rate can be represented as

$$\ln \frac{H_{t+1}}{H_t} = \ln \frac{(1-\eta n(h_{rt}))h_{rt}}{(1-\eta n(h_{rt-1}))h_{rt-1}} + \ln \frac{n(h_{rt-1})}{n_{p\mathrm{I}}}.$$

As the human capital level of the rich converges to its steady-state value, the growth rate of the ratio of skilled labor to unskilled labor becomes negative because of differential fertility:

$$\lim_{h_{rt}\to h^*} \ln \frac{H_{t+1}}{H_t} = \ln \frac{n(h^*)}{n_{p\mathrm{I}}} < 0.$$

Figure 4 shows the relationship between the ratio of skilled labor to unskilled labor and the threshold for the increase of the poor's income level, represented by H_{I}. The poor's income level can start to rise if the ratio of skilled labor to unskilled labor can exceed the threshold for the start. However, if the initial ratio of rich people to the total population is low, then a decrease in the ratio of the number of rich to poor people might outweigh an increase in the human capital level of the rich before the skilled labor to unskilled labor ratio exceeded the threshold for the start of the poor's income level. The start will become more difficult because of the decrease in the ratio of rich people to the total population if it takes time.

The existence of child labor increases the fertility rate of the poor. Thus, the existence of child labor strengthens the decrease in the ratio of the number of rich to poor people. Consequently, child labor makes it more difficult for the poor's income level to increase.[13] The lines $P_{\mathrm{I}s}$ and $P_{\mathrm{I}f}$ in Figure 4 represent, respectively, the success and failure conditions for the start of the poor's income

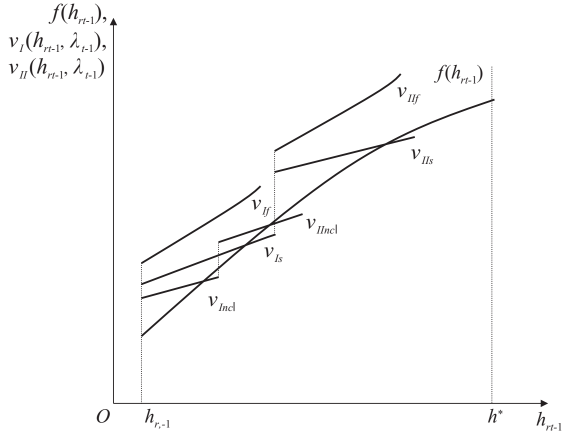

FIGURE 5. Conditions represented as (37) and (44).

level. The initial ratio of rich people to the total population is higher in $P_{\mathrm{I}s}$ than in $P_{\mathrm{I}f}$.

Next, we investigate (36) by dividing the ratio of skilled labor to unskilled labor into the human capital of the rich and the ratio of the number of rich to poor people. Using (30), (33), and (35), (36) can be rewritten as[14]

$$f(h_{rt-1}) > v_{\mathrm{I}}(h_{rt-1}, \lambda_{t-1}), \tag{37}$$

where

$$f(h_{rt-1}) \equiv [1 - \eta n(x(h_{rt-1}))]x(h_{rt-1}),$$

$$v_{\mathrm{I}}(h_{rt-1}, \lambda_{t-1}) \equiv H_{\mathrm{I}}(1 - \eta n_{pI} + b n_{pI}) \frac{n_{pI}}{n(h_{rt-1})} \frac{1 - \lambda_{t-1}}{\lambda_{t-1}}.$$

As the human capital of the rich increases, it converges to h^*. However, $v_{\mathrm{I}}(h_{rt-1}, \lambda_{t-1})$ also increases with the human capital accumulation of the rich; it asymptotically takes an infinite value. The poor's income level can start to increase if the human capital level of the rich exceeds $v_{\mathrm{I}}(h_{rt-1}, \lambda_{t-1})$. A low initial ratio of the number of rich to poor people implies a high initial value of $v_{\mathrm{I}}(h_{rt-1}, \lambda_{t-1})$. That large initial value makes it difficult for the poor's income level to increase. Furthermore, $v_{\mathrm{I}}(h_{rt-1}, \lambda_{t-1})$ increases more rapidly because of child labor. Therefore, it becomes more difficult for the poor's income level to increase because of child labor. Figure 5 shows the relationship between $f(h_{rt-1})$ and $v_{\mathrm{I}}(h_{rt-1}, \lambda_{t-1})$. The lines $v_{\mathrm{I}s}$ and $v_{\mathrm{I}f}$ in Figure 5 depict, respectively, the conditions for success and failure in the increase of the poor's income. The initial ratio of the number of rich to poor people is higher in $v_{\mathrm{I}s}$ than in $v_{\mathrm{I}f}$.

4.3. The Start of Educational Investment by the Poor

Assuming that (36) holds, i.e., the income level of poor people starts to rise, only modern technology is then used. Because traditional technology vanishes, child labor is unavailable. Using (17) and (18), which are the first-order conditions of firms using modern technology, we have

$$H_t^{-(1+\rho)} \frac{d}{1-d} = \frac{w_{st}}{w_{ut}}. \tag{38}$$

That is, the ratio of wage rates of unskilled labor to skilled labor starts to rise with an increase in the ratio of skilled labor to unskilled labor.

Although the poor's income level increases, they still may not start investment in education. We denote the stage before poor people start education investment as stage II. As poor people choose $q_{pt} = 0$, this implies that $e_{pt} = 0$. Because child labor is unavailable, the incentive to have children decreases. As shown in Figure 2, compared with $n_{p\mathrm{I}}$, the fertility rate decreases as[15]

$$n_{pt} = \frac{1-\beta}{\eta} \equiv n_{pII}. \tag{39}$$

The consumption level increases with an increase in the wage rate of unskilled labor:

$$c_{pt} = \beta w_{ut}. \tag{40}$$

The educational investment level of the rich and their fertility rate remain intact because neither quantity depends on the wage rate of skilled labor: no change exists in (27) and (28). The dynamics of the human capital level represented in (30) also remain intact. The consumption level is rewritten as

$$c_{rt} = \beta h_{rt-1} w_{st}. \tag{41}$$

Compared with stage I, the ratio of rich people to the total population decreases more slowly because of the decrease in the fertility rate of poor people:

$$\lambda_t = \frac{n(h_{rt-1})\lambda_{t-1}}{n(h_{rt-1})\lambda_{t-1} + n_{pII}(1-\lambda_{t-1})}. \tag{42}$$

Let us examine whether the poor can start to invest in education. Using (9) and (38), this condition is represented as

$$H_{t+1} > H_{II}, \tag{43}$$

where

$$H_{t+1} = \frac{(1-\eta n(h_{rt}))h_{rt}}{1-\eta n_{pII}} \frac{\lambda_t}{1-\lambda_t} \quad \text{and} \quad H_{\mathrm{II}} \equiv \left(\frac{o}{\delta\eta}\frac{d}{1-d}\right)^{1/(1+\rho)}.$$

If the ratio of skilled labor to unskilled labor exceeds the threshold for the start of education investment by poor people, represented by H_{II}, then the poor can start educational investment. However, the ratio of the number of rich to poor people is small at the start of stage II. Consequently, a decrease in the ratio of the number of rich to poor people might outweigh an increase in the human capital level of the rich before the ratio of skilled labor to unskilled labor exceeded the threshold represented by H_{II}. That is, if it takes time for a sufficient rise in the poor's income level, it will become more difficult for them to start education investment. The lines P_{IIs} and P_{IIf} in Figure 4 represent, respectively, the success and failure conditions for the start of educational investment by poor people. The ratio of the number of rich to poor people is higher in P_{IIs} than in P_{IIf}.

No direct effect of child labor is apparent because child labor has already vanished. However, when child labor is available in stage I, this availability makes the ratio of the number of rich to poor people smaller at the start of stage II. Therefore, the prior existence of child labor has a negative effect on the start of educational investment by poor people.

By dividing the ratio of skilled labor to unskilled labor into the human capital of the rich and the ratio of the number of rich to poor people, we rewrite the condition for poor people to start educational investment in (43) as

$$f(h_{rt-1}) > v_{\mathrm{II}}(h_{rt-1}, \lambda_{t-1}), \tag{44}$$

where

$$v_{II}(h_{rt-1}, \lambda_{t-1}) \equiv H_{II}(1 - \eta n_{pII}) \frac{n_{pII}}{n(h_{rt-1})} \frac{1 - \lambda_{t-1}}{\lambda_{t-1}}.$$

Compared with $v_{\mathrm{I}}(h_{rt-1}, \lambda_{t-1})$, the ratio of the number of poor to rich people increases less rapidly because of the lack of child labor. However, both this ratio and the human capital level of the rich have become high during stage I. A high ratio of the number of poor to rich people implies a high initial value of $v_{\mathrm{II}}(h_{rt-1}, \lambda_{t-1})$, which makes it more difficult for the poor to receive education. Figure 5 presents the relationship between $f(h_{rt-1})$ and $v_{\mathrm{II}}(h_{rt-1}, \lambda_{t-1})$. If the human capital level of the rich exceeds $v_{\mathrm{II}}(h_{rt-1}, \lambda_{t-1})$, then poor people can start educational investment. The lines $v_{\mathrm{II}s}$ and $v_{\mathrm{II}f}$ portray, respectively, the success and failure conditions for the start of educational investment by poor people.[16] The ratio of the number of rich to poor people is higher in $v_{\mathrm{II}s}$ than in $v_{\mathrm{II}f}$.

Finally, let us consider the effects of the prohibition of child labor on development. The prohibition of child labor decreases the fertility rate of poor people because of a lack of incentive to have more children. The ratio of rich people to the total population follows (42), but not (34), even in stage I, because the fertility rate of the poor in stage I is now represented as $n_{p\mathrm{II}}$, but not $n_{p\mathrm{I}}$. The ratio of the number of rich to poor people decreases less rapidly. Therefore, the ratio of skilled labor to unskilled labor increases more rapidly. The line P_{ncl} in Figure 4 shows the ratio of skilled labor to unskilled labor when child labor is prohibited in stage I. This line can more easily exceed the thresholds represented by H_I and H_{II}. Furthermore, the lines V_{Incl} and V_{IIncl} in Figure 5 show that, whereas $V_I(h_{rt-1}, \lambda_{t-1})$ increases less rapidly, $V_{II}(h_{rt-1}, \lambda_{t-1})$ increases less rapidly because of a lower ratio of the number of poor to rich people. It becomes less difficult for the poor's income level to increase. It also becomes less difficult for the poor to start educational investment.

5. CONCLUDING REMARKS

Demographic transition is important for further development. However, fertility remains high in less developed economies. Moreover, a large part of the population cannot escape poverty in those economies. Thus, this paper has considered differential fertility and its effect on development. Although it is necessary that the average human capital stock exceed certain thresholds for an escape from poverty, the race between the accumulation of human capital by rich people and the accumulation of children by poor people is crucial to the accumulation of the average human capital stock. If the ratio of the number of poor to rich people increases more rapidly than the human capital level of the rich, the average human capital stock will not rise, and thus, the poor will not be able to escape poverty.

The accumulation of the average human capital stock is more difficult when the ratio of the number of poor to rich people is high. Thus, if the initial ratio of the number of poor to rich people is high, it will be difficult for the poor to escape poverty. This relationship implies that the concentration of wealth in a small number of rich people might have an adverse effect on development. Additionally, child labor increases the fertility rate of the poor, making it more difficult for the poor's income level to increase. Even if child labor becomes unavailable, if the ratio of the number of poor to rich people is high because of the prior existence of child labor, then the poor will not be able to start educational investment. Therefore, if it takes time to abolish child labor, it will become more difficult for the poor to escape poverty.

NOTES

1. Benhabib and Spiegel (1994) found that the accumulation of the average human capital stock is important for growth in GDP per capita.

2. Assuming capital market imperfection, Galor and Zeira (1993) showed that educational opportunities can result in persistent inequality. Nakamura and Nakajima (2011) presented a theory that allowed poor people to invest by borrowing. They examined how a credit market helps relatively rich and poor people escape poverty.

3. Unified growth theories include Tamura (2002) and Doepke (2004). Tamura (2002) developed a model of economic and population growth that generates a transition between agriculture and industry. See Galor (2005), who extensively surveys long-run transition models.

4. Galor et al. (2009) showed that development becomes difficult with a large inequality in the distribution of land ownership. By assuming the average school enrollment ratio as an external effect, Momota (2009) showed that economic growth might slow even when the fertility transition starts.

5. The assumption that $0 < \delta < 1$ ensures the second-order conditions of the utility maximization problem when parents invest in education for their children. The assumption that $b < \eta$ ensures an inner solution of the fertility rate when parents force their children to work as child laborers.

6. Moav (2002) and Galor and Moav (2004) allowed a zero education expenditure with the assumption of altruistic bequest motives. If we assumed that parents care about the total income level of their children, it would be difficult to have ranges of the initial human capital level and parameter values satisfying some restrictions imposed in our model.

7. In Section 4.1, we revisit this assumption under the condition that rich people are skilled laborers.

8. This property is the same as that of de la Croix and Doepke (2003).

9. When $\rho = -1$, skilled and unskilled labor are perfectly substitutable. Because the wage rates of skilled and unskilled labor will always be constant, the poor can never escape poverty.

10. We assume that $A_T > A_M(1-d)^{-1/\rho}$, where $-1 < \rho < 0$, to avoid the case in which modern technology always dominates traditional technology.

11. Even if the human capital stock of rich people increased unboundedly in our model, development would still depend on a race between the accumulation of human capital by rich people and the accumulation of children by poor people. Becker et al. (1990) and Tamura (1996) considered no growth with high fertility and little human capital and sustained growth with low fertility and rising human capital.

12. The population growth rate, represented as L_{t+1}/L_t, is equal to $\lambda_t n(h_{rt-1}) + (1-\lambda_t)n_{pI}$.

13. Development also depends on technologies. Because a high productivity of traditional technology increases H_{I}, development becomes more difficult. In Appendices A and B, we examine the effect of the elasticity of substitution between skilled and unskilled labor on development.

14. In Appendix C, we present the explanation of (37) in detail.

15. The fertility rate represented by $n_{p\mathrm{II}}$ is equivalent to that expressed in (12) when $\delta\eta w_{ut} = o w_{st}$ holds. The assumption that $1-\beta > \eta$ ensures that the case in which the total population always decreases is avoided.

16. If (43) holds, only skilled laborers exist. Because the dynamics of the human capital levels of the rich and the poor are identical, income inequality between the rich and poor will disappear in the long run.

17. Klump and de La Grandville (2000), Klump and Preissler (2000), Miyagiwa and Papageorgiou (2003), and Nakamura (2009, 2010) used this procedure to investigate the exact relationship between economic growth and the elasticity of substitution between capital and labor.

18. This proof is available on request. Our proof is essentially the same as that of Miyagiwa and Papageorgiou (2003).

19. Although the initial human capital level and parameters must satisfy (25), (26), the assumptions in (30) and (C.4), and footnotes 5, 10, and 15, there exist ranges of those values.

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APPENDIX A

We examine the effect of the elasticity of substitution between skilled and unskilled labor on the start of a rise in the poor's income level. We use the normalization procedure for a CES production function developed by de La Grandville (1989) to examine the exact effects of the elasticity of substitution between skilled and unskilled labor on development.[17] We arbitrarily choose baseline values for three variables: the ratio of skilled labor input to unskilled labor input, $\bar{H}$; output per labor unit, $\bar{y}$; and the marginal rate of substitution, $\bar{m} \equiv (\partial Y_t/\partial l_{Mut})/(\partial Y_t/\partial l_{Mst})$, which is evaluated at $\bar{H}$. We can then obtain the normalized distribution parameter and the normalized efficiency parameter as functions of the elasticity of substitution, given $\bar{H}$, $\bar{y}$, and $\bar{m}$. The CES production function can be represented as

$$Y_t = A_M(\sigma)\{d(\sigma)l_{Mst}{}^{-\rho} + [1-d(\sigma)]l_{Mut}{}^{-\rho}\}^{-1/\rho}, \quad \textbf{(A.1)}$$

where

$$d(\sigma) \equiv \frac{\bar{H}^{1+\rho}}{\bar{H}^{1+\rho}+\bar{m}} \quad \text{and} \quad A_M(\sigma) \equiv \bar{y}\left(\frac{\bar{H}^{1+\rho}+\bar{m}}{\bar{H}+\bar{m}}\right)^{-1/\rho}.$$

From (16) and (18), we define the wage rate of unskilled labor evaluated at H_{I} as follows:

$$g(H_{\mathrm{I}} : \sigma) \equiv A_M(\sigma)\{d(\sigma)H_{\mathrm{I}}^{-\rho} + [1-d(\sigma)]\}^{-(1+\rho)/\rho}[1-d(\sigma)] = w_{u0}. \quad \textbf{(A.2)}$$

The differentiation of (A.2) with respect to the elasticity of substitution between skilled labor and unskilled labor is written as follows:

$$\frac{\partial g(H_I : \sigma)}{\partial \sigma}|_{\text{given } H_I} + \frac{\partial g(H_I : \sigma)}{\partial H_I}\frac{\partial H_I}{\partial \sigma} = 0. \quad \textbf{(A.3)}$$

If $-\rho \geq \bar{m}/\bar{H}$ holds, we have $\partial g(H_I : \sigma)/\partial \sigma|_{H_{\mathrm{I}}\text{given}} < 0$ for any $H_I > \bar{H}$.[18] That is, when skilled and unskilled labor are relatively substitutable, a higher elasticity of substitution implies that the wage rate of unskilled labor increases less rapidly. Furthermore, we have $\partial g(H_I : \sigma)/\partial H_I > 0$ because the marginal product of unskilled labor increases with an increase in the ratio of the skilled labor input to the unskilled labor input. Therefore, (A.3) implies that $\partial H_I/\partial \sigma > 0$ holds for any $H_I > \bar{H}$ as long as $-\rho \geq \bar{m}/\bar{H}$ holds. That is, a relatively high elasticity of substitution implies a high threshold level of the skilled labor/unskilled labor ratio for the rise of the poor's income level.

APPENDIX B

We investigate the effect of the elasticity of substitution on whether poor people can start educational investment. They can start accumulating their human capital if $H_{t+1} > H_{II}$ holds. This condition is equivalent to $\eta\delta w_{ut} > o w_{st}$. Using the first-order conditions of the cost minimization problem of firms, we have

$$q(H_t : \sigma) \equiv H_t^{-(1+\rho)}\frac{d(\sigma)}{1-d(\sigma)} = \frac{w_{st}}{w_{ut}}. \quad \textbf{(B.1)}$$

The differentiation of (B.1) with respect to the elasticity of substitution between skilled labor and unskilled labor is written as

$$\frac{\partial q(H_t : \sigma)}{\partial \sigma} = \frac{\partial q(H_t : \sigma)}{\partial \sigma}|_{\text{given } \mathrm{H_t}} + \frac{\partial q(H_t : \sigma)}{\partial H_t}\frac{\partial H_t}{\partial \sigma} = \frac{\partial q(H_t : \sigma)}{\partial \sigma}|_{\text{given } \mathrm{H_t}}. \quad \textbf{(B.2)}$$

The human capital level of the rich and the fertility rates of the rich and the poor are not affected by the wage rates, i.e., the elasticity of substitution. Thus, we have $\partial H_t/\partial \sigma = 0$. We also obtain

$$\frac{\partial \ln q(H_t : \sigma)}{\partial \rho}|_{\text{given } \mathrm{H_t}} = -\ln\frac{H_t}{\bar{H}} < 0,$$

for any $H_t > \bar{H}$. Furthermore, $\partial\rho/\partial\sigma < 0$ holds. We then obtain

$$\frac{\partial q(H_t : \sigma)}{\partial\sigma}\Big|_{\text{given } H_t} > 0.$$

A higher elasticity of substitution implies that the ratio of the wage rates of unskilled labor to skilled labor increases less rapidly. Thus, it becomes more difficult for the poor to start education investment.

APPENDIX C

Let us investigate (37) in more detail. Given λ_{t-1}, we consider the equality between $f(h_{rt-1})$ and $v_I(h_{rt-1}, \lambda_{t-1})$ at $\hat{h}_{rt-1}$,

$$f(\hat{h}_{rt-1}) = k(\hat{h}_{rt-1})\frac{1-\lambda_{t-1}}{\lambda_{t-1}}, \tag{C.1}$$

where $k(h_{rt-1}) \equiv H_I(1 - \eta n_{pI} + bn_{pI})\frac{n_{pI}}{n(h_{rt-1})}$.

By differentiating (C.1) with respect to h_{rt-1}, we also consider the following equality at $\hat{h}_{rt-1}$:

$$f'(\hat{h}_{rt-1}) = k'(\hat{h}_{rt-1})\frac{1-\lambda_{t-1}}{\lambda_{t-1}}. \tag{C.2}$$

Figure C.1 illustrates the equality noted in (C.1) and (C.2).

Using (C.1) and (C.2), we obtain the following:

$$\frac{f(\hat{h}_{rt-1})}{f'(\hat{h}_{rt-1})} = \frac{k(\hat{h}_{rt-1})}{k'(\hat{h}_{rt-1})}. \tag{C.3}$$

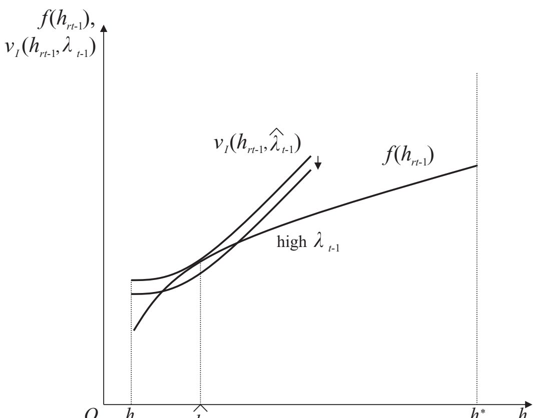

FIGURE C.1. Condition represented as (C.3).

We define the following function:

$$F(h_{rt-1}) \equiv \frac{k(h_{rt-1})}{k'(h_{rt-1})} / \left[\frac{f(h_{rt-1})}{f'(h_{rt-1})} \right].$$

We can assure $F'(h_{rt-1}) > 0$ by imposing some restrictions on parameters.

We assume that[19]

$$F(h_{r,-1}) < 1 \quad \text{and} \quad F(h^*) > 1. \tag{C.4}$$

There then exists $\hat{h}_{rt-1}$ such that $F(\hat{h}_{rt-1}) = 1$.

By using (C.1), we define $\hat{\lambda}_{t-1}$ at $\hat{h}_{rt-1}$. We then have the intersection between $f(h_{rt-1})$ and $v_{\mathrm{I}}(h_{rt-1}, \lambda_{t-1})$ as long as $\lambda_{t-1} > \hat{\lambda}_{t-1}$ holds, because $v_{\mathrm{I}}(h_{rt-1}, \lambda_{t-1})$ is a decreasing function with respect to λ_{t-1} and $f(h_{rt-1})$ does not depend on λ_{t-1}. That is, (37) can hold as the ratio of the rich to the total population takes a high value.